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UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WASHINGTON

August 10, 1945.

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WATER RESOURCES BULLETIN

A memorandum for the official use of members of the Water Resources Branch, Geological Survey.

By direction of the Secretary of the Interior the matter contained herein is presented as administrative information and is required for the proper transaction of public business.

As the contents are highly informative and relate generally to current work, the Bulletin should be made available as promptly as practicable to each member of the technical personnel of the Branch. The topics should be studied and, as there is opportunity, thoroughly discussed by all. The views expressed and the statements made in this Bulletin are those of the writers and not necessarily those of the district offices to which the writers are assigned, or those of the Washington office.

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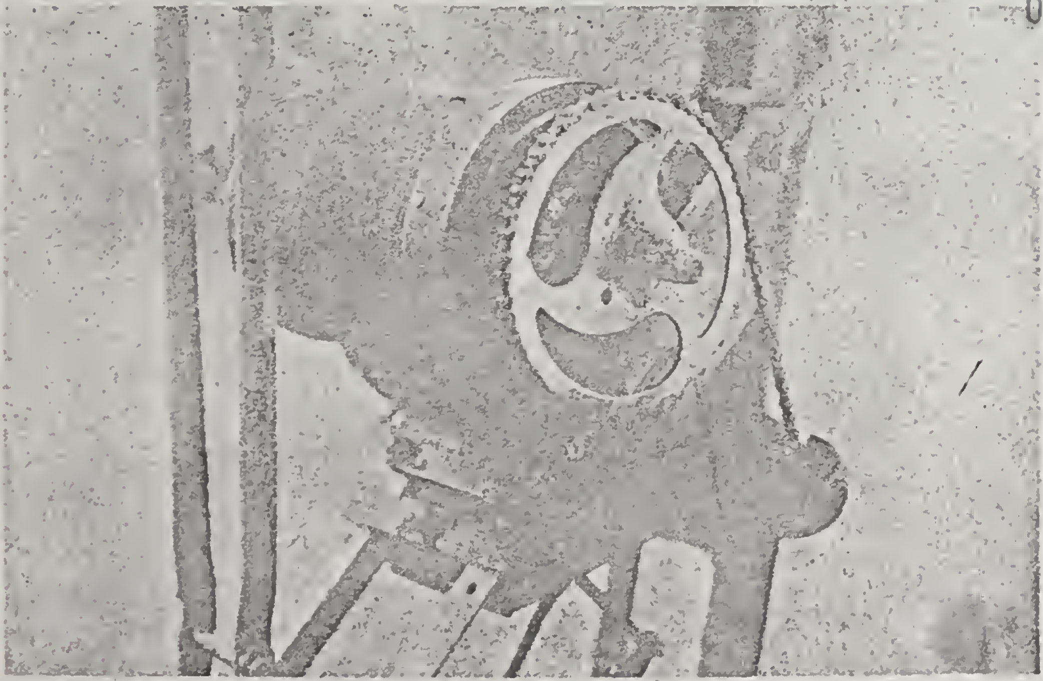
BATTERY-OPERATED SOUNDING REEL
(See page 136)



Photograph by E. B. Rice, June 8, 1945

WELLS

Partial penetration of pumping well, adjustments for, by C. E. Jacob, hydraulic engineer, Division of Ground Water, Washington office.

Synopsis

The partial penetration of a pumping well is important, not only because it affects the efficiency of the well but also because it influences the distribution of head in its vicinity, modifying the drawdowns observed in nearby observation wells. In running pumping tests to determine transmissibility, the observed drawdowns must be adjusted for this factor. This article outlines the method of making the necessary adjustments. Graphs and a table are given for partial penetration in homogeneous artesian beds, supplementing the cursory treatment given this problem in Water-Supply Paper 887.

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The partial penetration of a pumping well is a matter of great practical interest. This is true first because the productivity of a well depends largely on the degree to which it penetrates a water-producing bed. Second, the distribution of head in the vicinity of a partially penetrating well is modified according to the degree of penetration, and therefore water levels observed in nearby observation wells during pumping tests should be adjusted for that effect before an attempt is made to determine the transmissibility or permeability.

The problem of a partially penetrating well was analyzed in great detail by Muskat in 1932. (See Physics, vol. 2, pp. 329-364; also "The flow of homogeneous fluids through porous media," McGraw-Hill, 1937, pp. 263-286.) He gave an equation for the discharge of a partially penetrating well in terms of sand-thickness, permeability, fractional penetration, and total potential-drop from the assumed effective external radius of the system to the known internal radius. (See equation 6, p. 274 of Muskat's book.) In 1933 Kozeny (Wasserkraft und Wasserwirtschaft, vol. 28, p. 101) published a somewhat simpler empirical expression summarizing Muskat's analysis to a sufficient approximation formany purposes. In our notation Kozeny's empirical formula is

$$Q = 2\pi T s_w \alpha \left[1 + 7(r_w/2\alpha m)^{1/2} \cos(\pi\alpha/2)\right] / \log_e(r_e/r_w) \qquad (1)$$

or

$$Q = Q_o \alpha \left[1 + 7(r_w/2\alpha m)^{1/2} \cos(\pi\alpha/2) \right. = Q_o \alpha/C \qquad (2)$$

Here Q is the discharge of a well of radius r_w penetrating a fraction of a bed of thickness m and transmissibility T; Q_o is the discharge of a well of the same effective radius penetrating the bed completely. The drawdown in the well (i.e., at a distance r_w) is s_w. The drawdown at the virtual external radius of the system, r_e, is zero. The quantity in brackets is abbreviated as (1/C).

The apparent transmissibility, referred to the total thickness of the bed and determined from observations of the drawdown in a pumping well, is given by

$$T' = Q \log_e(r_e/r_w)/2\pi s_w \tag{3}$$

Similarly the true transmissibility is

$$T = Q_o \log_e(r_e/r_w)\ /2\pi s_w \tag{4}$$

$$T/T' = Q_o/Q = C/\alpha \tag{5}$$

The corresponding ratio of true to apparent permeability, the latter being referred, however, to an equivalent bed of thickness αm instead of to the total thickness, is

$$\alpha Q_o/Q = C. \tag{6}$$

This "correction factor" is given by Wenzel (Water-Supply Paper 887, p. 109, equation 124). It should be emphasized that this "correction" or adjustment is only for values of permeability determined from the drawdown observed *inside a pumping well*. Moreover, equation 6 (or Wenzel's equation 124) does not lead to a solution of the problem, as Q_o is *unknown*. Actually Q_o may be determined from Muskat's equation 5.4(6), or it may be determined from equation 2 above.

Wenzel quotes Muskat as stating that the flow toward a partially penetrating well will become almost exactly radial at a distance from the well equal to twice the sand-thickness. (See Water-Supply Paper 887, p. 110; Muskat, p. 283.) It should be pointed out, however, that this applies to *isotropic beds only*. Most water-lain beds are stratified and therefore anisotropic, taken as a whole. Even homogeneous alluvial materials are anisotropic as regards permeability.

The critical distance referred to is to be multiplied by the square root of the ratio of horizontal to vertical permeability. For example, if a bed is on the average sixteen times as permeable in the horizontal as in the vertical direction, then purely radial flow occurs only beyond a distance equal to about 8 times the sand-thickness. Accordingly, care should be taken to adjust drawdowns observed within that distance in determining the transmissibility in the case cited.

In order to make the adjustment for partial penetration it is necessary to know the distribution of head *throughout the bed*. (The foregoing relations are inappropriate because they relate merely to the head-difference between the inflow and outflow surfaces.) The head-distribution about a partially penetrating well is given by equation 5.3(8) or equation 5.3(9) of Muskat's book. The first mentioned equation is convenient for computations for small values of r; the second for large values of r. Equation 5.3(9) contains two terms, one for the logarithmic distribution of head for purely radial flow, the other for the difference between the actual distribution and the logarithmic distribution. For convenience we may restrict our attention to the distribution of head along the top of the bed and along the bottom of the bed, such as given by drawdown readings in one series of observation wells

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by C. E. Jacob

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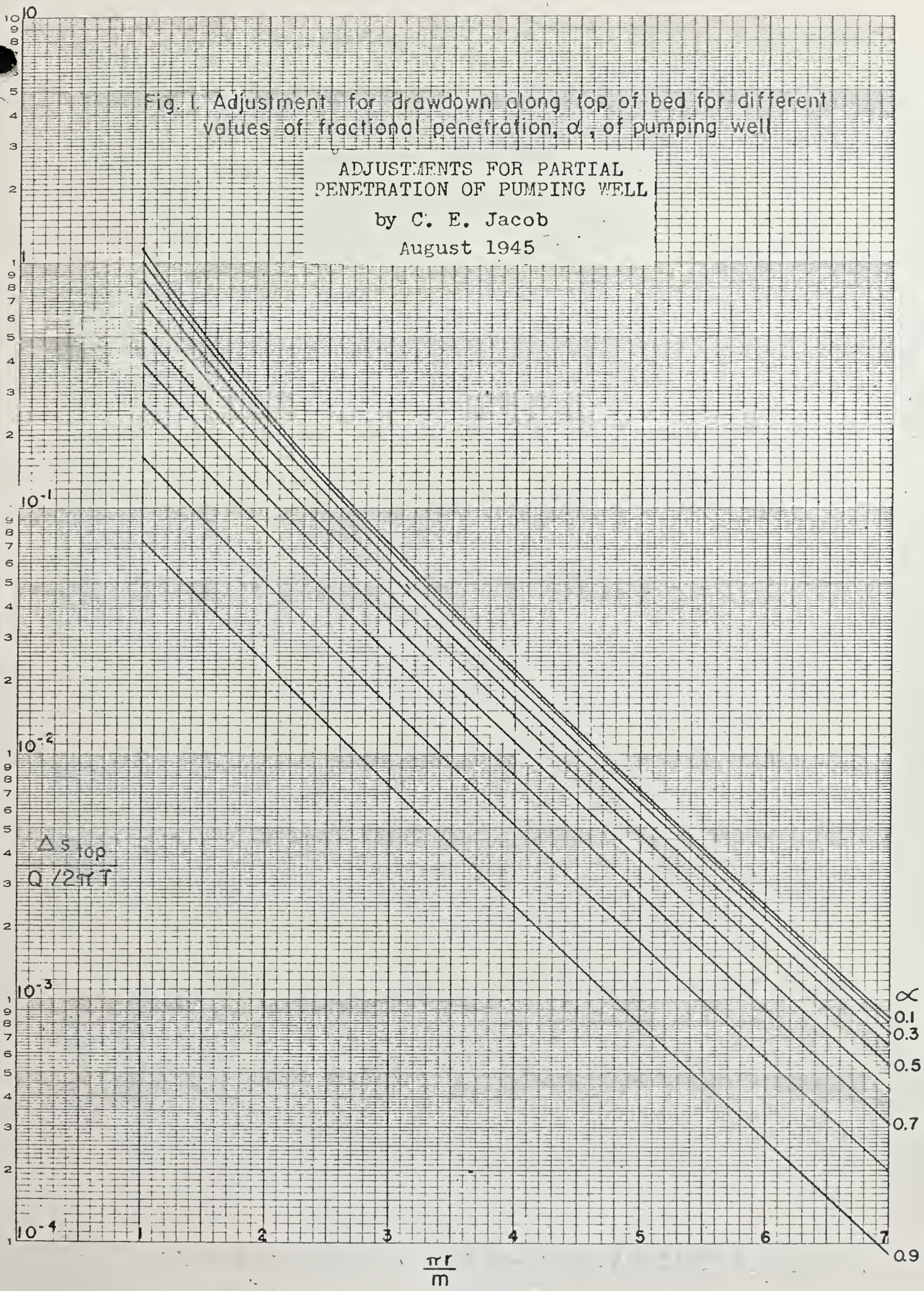

Fig. 1. Adjustment for drawdown along top of bed for different values of fractional penetration, α, of pumping well

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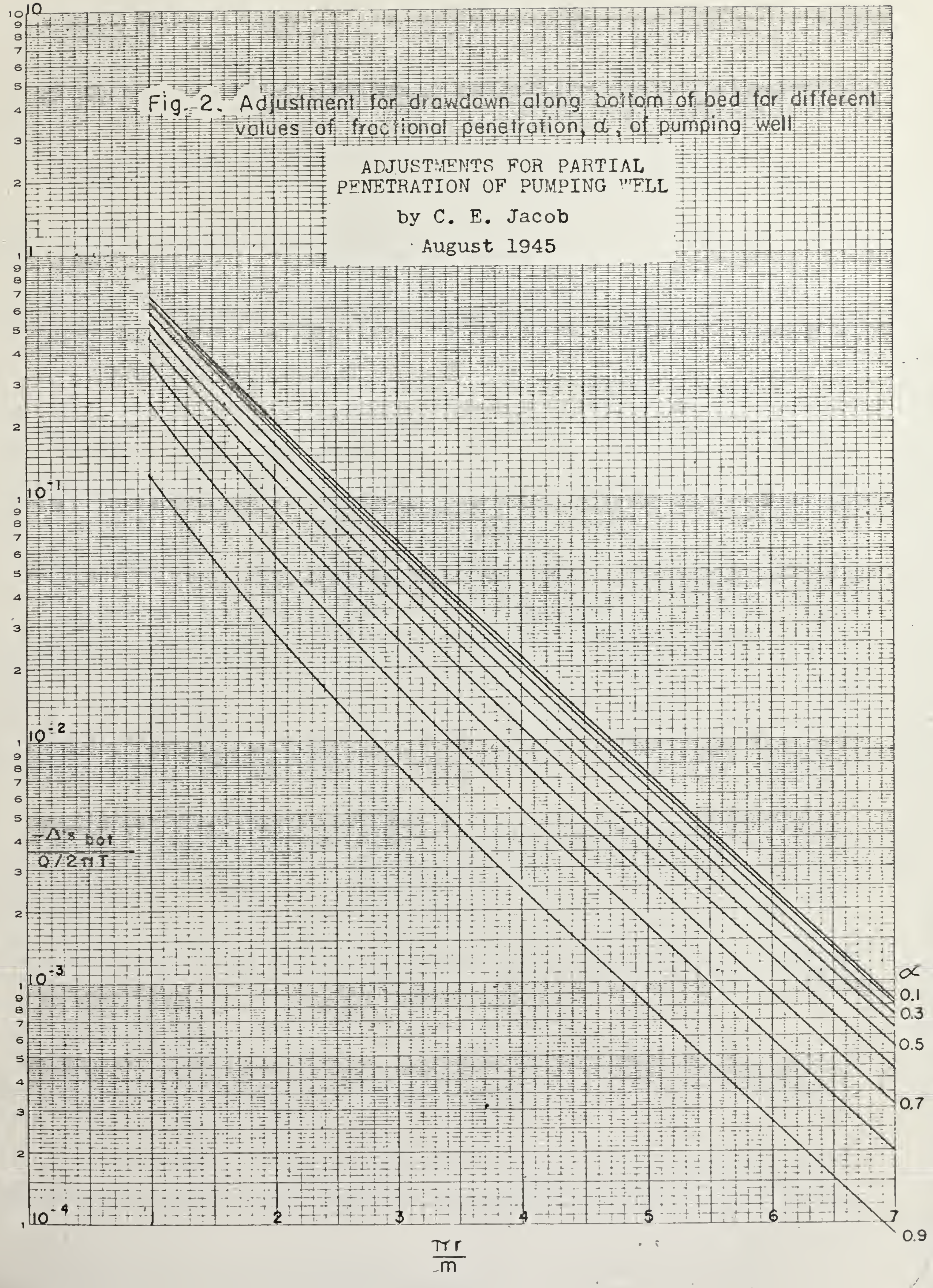

Fig. 2. Adjustment for drawdown along bottom of bed for different values of fractional penetration, α, of pumping well

or piezometers just reaching to the top of the bed and another reaching to the bottom of the bed and open only at that level. The divergence of the head from a pure logarithmic distribution at a distance r from the well is given by

$$\Delta s/(Q/2\pi T) = (2/\pi\alpha)\sum_{n=1}\left[(\pm 1)^n K_o(n\pi r/m)\sin(n\pi\alpha)\right]/n \qquad (7)$$

the plus sign being taken for the drawdown distribution along the top of the bed, the minus sign for the bottom of the bed.

In equation 7 the K_o stands for the modified Bessel function of the second kind of zero order. The other symbols have the same meanings as were assigned previously.

Figures 1 and 2 were computed from equation 7. Figure 1 gives the drawdown adjustment for the top of the bed, that is, the difference between the drawdown at the top of the bed, at a given distance for different values of fractional penetration, and the drawdown at the same point with complete penetration of a well discharging at the same rate. Similarly, figure 2 gives the drawdown adjustment for the bottom of the bed, or the difference between the drawdown at the bottom of the bed, at a given distance for different values of fractional penetration, and the drawdown at the same point with complete penetration of the pumping well, the discharge again remaining fixed.

It is assumed that the well penetrates the bed partially from the top. In that case, because of the convergence of the stream-lines upon the screen, the drawdown will obviously be greatest along the top of the bed and least along the bottom. If the well is screened only at the bottom of the bed the problem is similar though inverted, assuming of course that no part of the bed is unwatered. If the screen is in some intermediate position the curves in figures 1 and 2 do not apply. Nor do they apply, strictly speaking, to unconfined beds.

An example may serve to illustrate the application of the drawdown adjustments. Assume that a well with an effective radius of 1.0 foot and with a screen set in the top 40 feet of an artesian bed 100 feet thick has been pumping at a constant rate of 840,000 g.p.d. long enough to establish steady radial flow beyond the most distant of three observation wells. The observation wells are respectively 50, 100, and 150 feet from the pumping well. They just tap the top of the bed, as shown in figure 4. The drawdowns observed in those wells are 5.02 feet, 3.18 feet, and 2.30 feet respectively, as indicated by the open circles plotted in figures 3 and 4. The drawdown in the pumping well is 18.4 feet. The screen-loss is negligible.

Assume that the bed is homogeneous and isotropic as regards permeability. The problem is to determine the transmissibility from the observed drawdowns. The procedure is as follows:

First, plot the drawdowns against the logarithm of the distance, as on figure 3. Draw a straight line (line I) as closely as possible through the points plotted. Make a preliminary determination of transmissibility, using the relation

$$T = 2.30\ Q/2\pi\Delta s \qquad (8)$$

where Δs is the drawdown difference over one log cycle.

Substituting, one gets

$$T = 2.30 \times 840{,}000 \text{ gpd}/2\pi(9.1 - 3.2) \text{ ft} = 52{,}000 \text{ gpd/ft},$$

and

$$Q/2\pi T = \Delta s/2.30 = 5.9 \text{ ft}/2.30 = 2.56 \text{ ft}.$$

Assuming, as a first approximation, that the above value of T is correct, make the following adjustments, remembering that $\alpha = 0.40$.

Well	1	2	3
Distance, r, ft	50	100	150
$\pi r/m$	1.57	3.14	4.71
Observed s_{top}, ft	5.02	3.18	2.30
Adjustment, from figure 1	0.31	0.045	0.008
Δs_{top}, ft	0.80	0.12	0.02
Adjusted drawdown, ft	4.22	3.06	2.28

Plot the new values of adjusted drawdown (x's on figure 3). Draw a straight line through the points (line II). Make a second trial determination of T:

$$T = 2.30 \times 840{,}000 \text{ gpd}/2\pi(7.1 - 3.1) \text{ ft} = 77{,}000 \text{ gpd/ft},$$

and

$$Q/2\pi T = \Delta s/2.30 = 4.0 \text{ ft}/2.30 = 1.74 \text{ ft}.$$

Readjust the observed drawdowns, proceeding as before:

Well	1	2	3
Observed s_{top}, ft	5.02	3.18	2.30
Adjustment, from figure 1	0.31	0.045	0.008
Δs_{top}, ft	0.54	0.08	0.01
Adjusted drawdown, ft	4.48	3.10	2.29

Again plot the new values of adjusted drawdown (+'s on figure 3). Draw a straight line through the points (line III). Make a third trial determination:

$$T = 2.30 \times 840{,}000 \text{ gpd}/2\pi(7.7 - 3.1)\text{ft} = 67{,}000 \text{ gpd/ft},$$

and

$$Q/2\pi T = \Delta s/2.30 = 4.6 \text{ ft}/2.30 = 2.00 \text{ ft}.$$

Carrying the procedure through a fourth time one gets line IV, from which T = 69,000 gpd/ft and $Q/2\pi T$ = 1.94 ft. This is close enough to the probable true value. Further computations are not called for.

As a further check, however, one may determine the drawdown that would occur in a completely penetrating well from Kozeny's formula (equation 1). Solving that equation for $(Q/2\pi T)\log_e(r_e/r_w)$ and substituting in it the proper values, one should get a

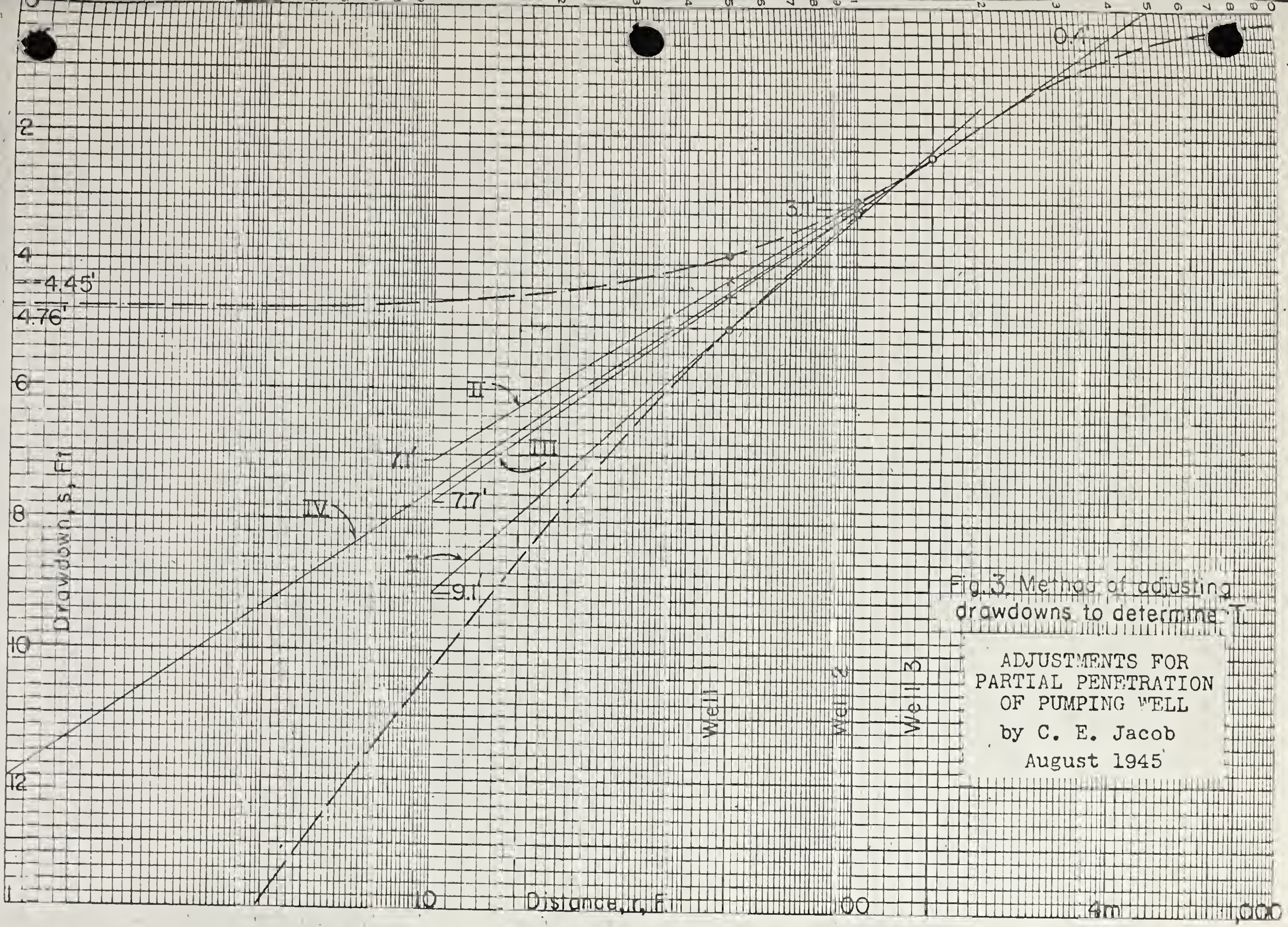

Fig. 3 Method of adjusting drawdowns to determine T

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drawdown that is checked by the intercept of line IV on the line $r = r_w = 1$ ft in figure 3. The hypothetical drawdown comes out to be $(s_w \alpha / C)$. In this case,

$$(1/C) = 1 + 7\left[\cos(0.2\pi)\right]/\sqrt{80} = 1 + \left[7(\cos 0.628)/8.94\right] = 1.634$$

and therefore

$$(s_w \alpha/C) = 18.4 \text{ ft} \times 0.40 \times 1.634 = 12.0 \text{ ft},$$ which is verified in figure 3.

The lower dashed line in figure 3 gives the probable distribution of drawdown along the top of the bed. The drawdowns that would have been observed in wells 1, 2, and 3 had they reached to the bottom of the bed and had they been open only at that level may be determined by making a further adjustment, using figure 2:

Well	1	2	3
Adjusted drawdown, ft (from line IV, figure 3)	4.42	3.09	2.28
Adjustment, from figure 2	-0.28	-0.044	-0.008
Δs_{bot}, ft	-0.54	-0.09	-0.01
s_{bot}, ft	3.88	3.00	2.27

For wells 1 and 2 these points are plotted as closed circles on figure 3. The distinction could not be made in plotting the data for well 3 and therefore that closed circle is omitted.

The upper dashed line in figure 3 gives the probable distribution of drawdown along the bottom of the bed. The drawdown at the point of stagnation underlying the well may be determined quite readily to aid in drawing the drawdown-distribution curve.

Table 1 gives the difference between the drawdown at the point of stagnation and the drawdown at a distance of 4m in units of $(Q/2\pi T)$ for different values of α under two alternative assumptions regarding conditions at the well-face. The data in the second column were computed by the writer from Muskat's equation 5.3(8), which is based upon the assumption that the flux through the face of the well, that is, the screen-velocity, is uniform. The data in the third column were made available to the writer through the courtesy of Dr. Muskat and the Gulf Research and Development Company. They were obtained by adjusting, more or less by cut and try, the flux-density distribution so as to give essentially uniform potential distribution over the well-face.

Table 1.

Fractional penetration α	Drawdown at point of stagnation underlying well $(s_o - s_{4m})/(Q/2\pi T)$	
	for uniform screen-velocity	for uniform drawdown at screen
0	(1.96)	
0.05		2.34
.10	1.971	2.25
.20	1.992	
.25		2.15
.30	2.030	
.40	2.087	(2.25)a
.50	2.170	2.37
.60	2.287	
.70	2.459	
.75		2.80
.80	2.733	
.90	3.264	3.70
1.00	∞	∞

a) by interpolation

In the present problem 4m = 400 feet. At that distance the drawdown, taken from line IV in figure 3, is 0.40 ft. Depending on which of the assumptions is chosen one gets either of the following values for the drawdown at the point of stagnation:

assuming uniform screen-velocity--

$$s_o = 2.09(Q/2\pi T) + 0.40 \text{ ft} = (2.09 \times 1.94 \text{ ft}) + 0.40 \text{ ft} = 4.45 \text{ ft}.$$

assuming uniform drawdown at screen--

$$s_o = 2.25(Q/2\pi T) + 0.40 \text{ ft} = (2.25 \times 1.94 \text{ ft}) + 0.40 \text{ ft} = 4.76 \text{ ft}.$$

The latter assumption probably fits actual conditions more closely than the former and has been adopted for figure 3. However, if the pump intake is above the top of the screen, owing to the friction of the upward moving water inside the screen the drawdown will be greatest at the top of the screen and least at the bottom. Accordingly, the screen-velocity may actually be more nearly uniform than it is when the drawdown is uniform. In other words, actual conditions probably lie somewhere between the two limits set by these assumptions.

In homogeneous beds the drawdown at the point of stagnation theoretically is independent of the ratio of horizontal to vertical permeability, provided that ratio does not become infinite. Radial distances are to be divided by the square root of that ratio before making adjustments in the drawdown at other points. (In the present problem that ratio is taken as unity.)

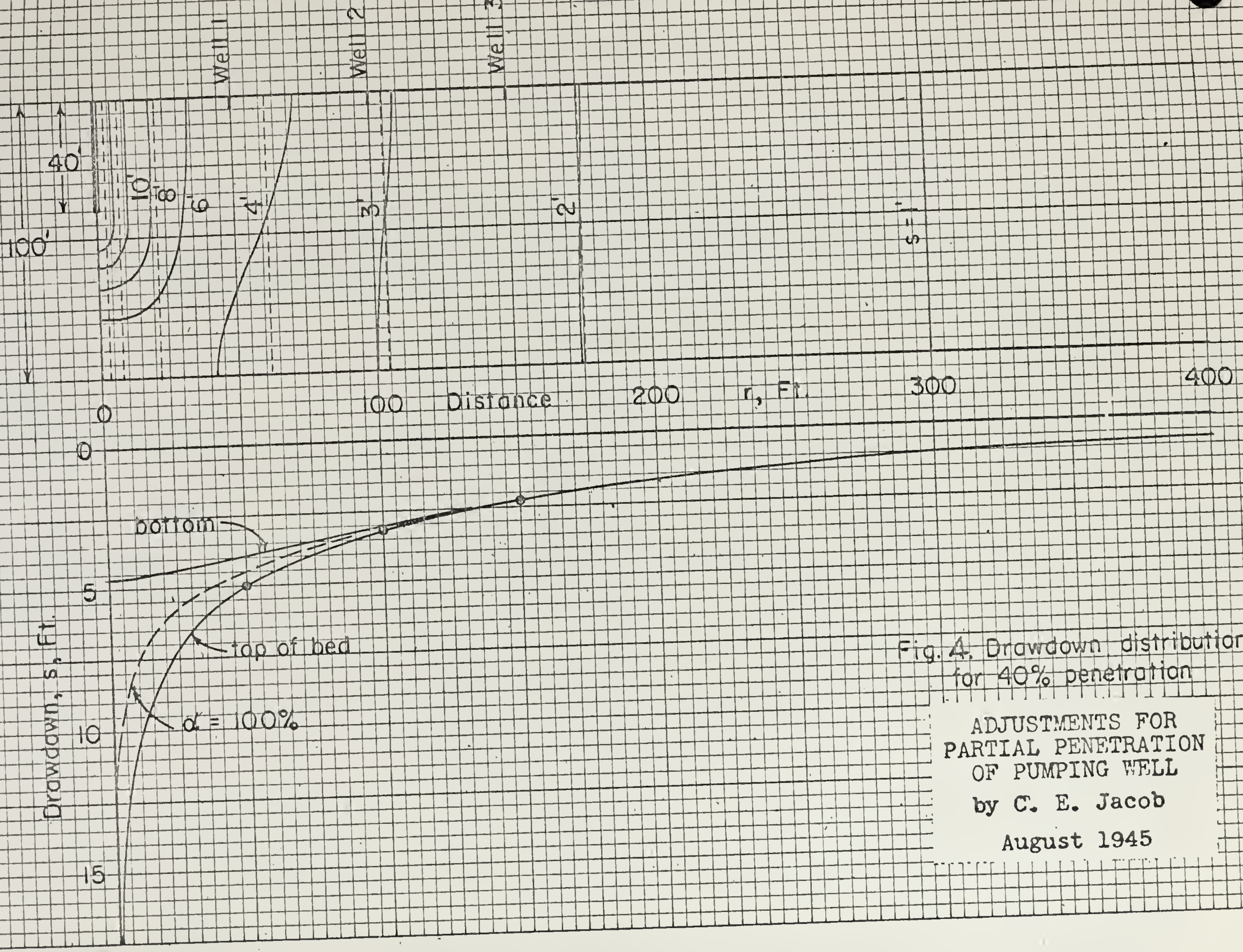

Fig. 4. Drawdown distribution for 40% penetration

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Figure 4 gives the approximate drawdown distribution along the top and bottom of the bed and throughout the bed for 40 percent penetration. The equipotentials (lines of equal drawdown) in that figure are to be regarded as diagrammatic, having been sketched in by hand from known values of drawdown along the top and bottom of the bed.

It may be seen that by averaging drawdowns observed at the top and bottom of the bed the approximate drawdown for complete penetration is obtained. The data from the foregoing example bear this out:

Well	1	2	3
Observed s_{top}, ft	5.02	3.18	2.30
Computed s_{bot}, ft	3.88	3.00	2.27
Final adjusted s, ft	4.42	3.09	2.28
Avg. s_{top} and s_{bot}, ft	4.45	3.09	2.28+

The difference is well within the error of plotting in this case. For 50 percent penetration the drawdowns average out exactly.

It is recommended that in running pumping tests to determine T with partially penetrating wells the observation wells be placed in pairs, one of each pair tapping the top of the bed, the other the bottom.

A remark about Slichter's formula for a partially penetrating well may not be out of place here. Reference is made to Wenzel's equation (123) Water-Supply Paper 887, p. 109). That formula is unsound because it assumes radial flow superimposed upon hemispherical flow, it being impossible to adjust the potential distribution satisfactorily along their common boundary.

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